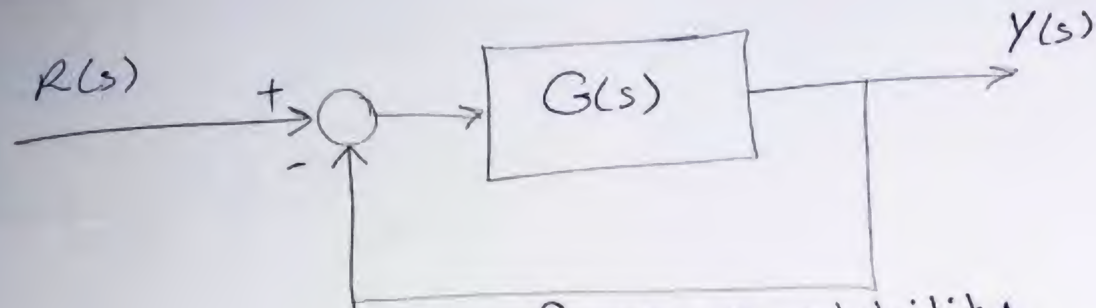
Lec 14 نظم واشارات

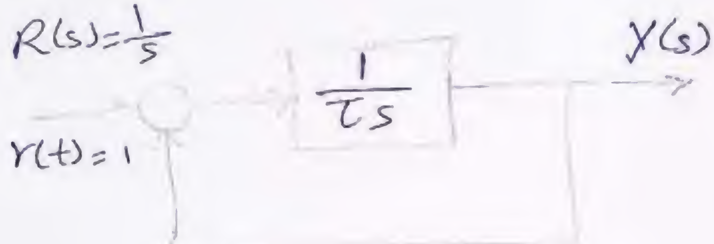
* stability of Control systems :-



unity feedback stability

* unit step response:-

1 st order system:-



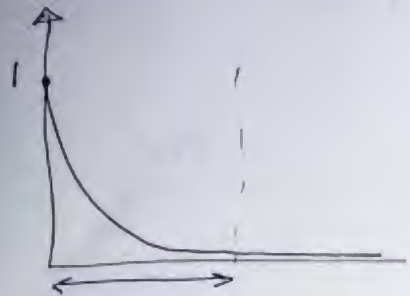
$$\frac{Y(s)}{R(s)} = \frac{1}{1+\tau s} = \frac{\frac{1}{\tau}}{s+\frac{1}{\tau}}$$

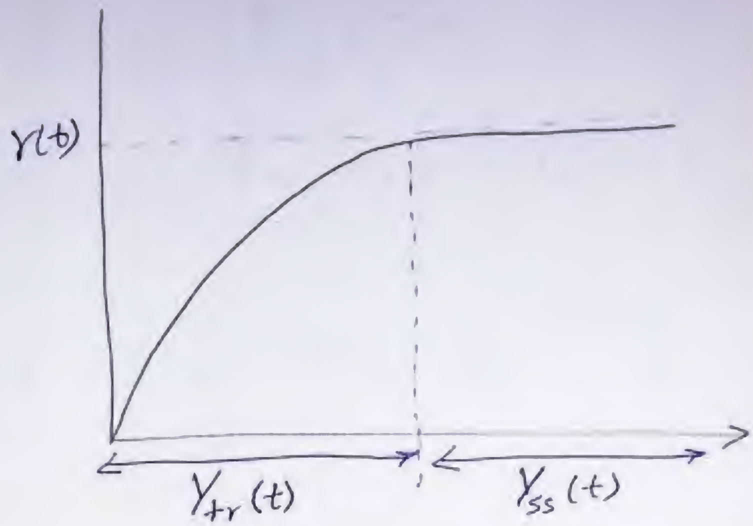
$$Y(s) = \left(\frac{\frac{1}{\tau}}{s+\frac{1}{\tau}}\right) R(s) = \left[\frac{\frac{1}{\tau}}{s+\frac{1}{\tau}}\right] * \frac{1}{s}$$

$$y(t) = 1 - e^{-t/\tau} = Y_{ss}(t) + Y_{tr}(t)$$

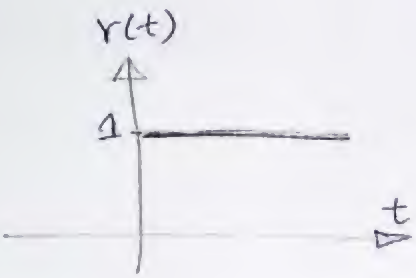
$$Y(s) = \frac{1}{s} \cdot \frac{-1}{s+\frac{1}{\tau}}$$

$$Y_{tr}(t) = e^{-t/\tau}$$



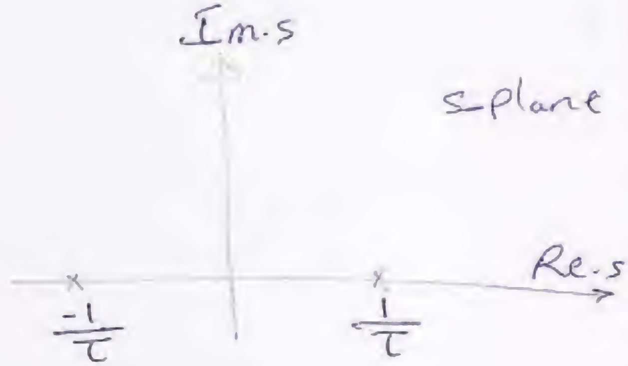


bounded input



stable system

Poles $\frac{-1}{\tau}$



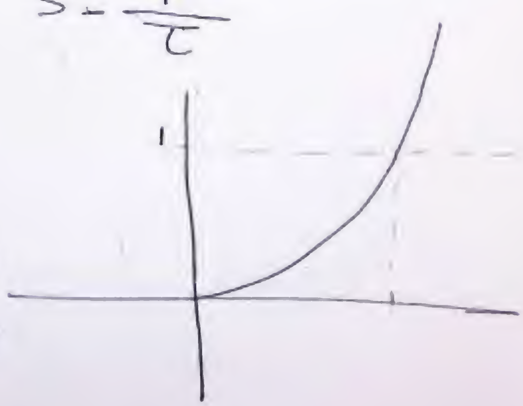
$$\frac{Y(s)}{R(s)} = \frac{1}{1-\tau s}$$

$$Y(s) = \frac{\frac{-1}{\tau}}{s-\frac{1}{\tau}}\left(\frac{1}{s}\right) = \frac{1}{s}\cdot\frac{-1}{s-\frac{1}{\tau}}$$

$$y(t) = 1 - e^{-t/\tau}$$

unbounded

unstable



* From unity Feed back stability:-

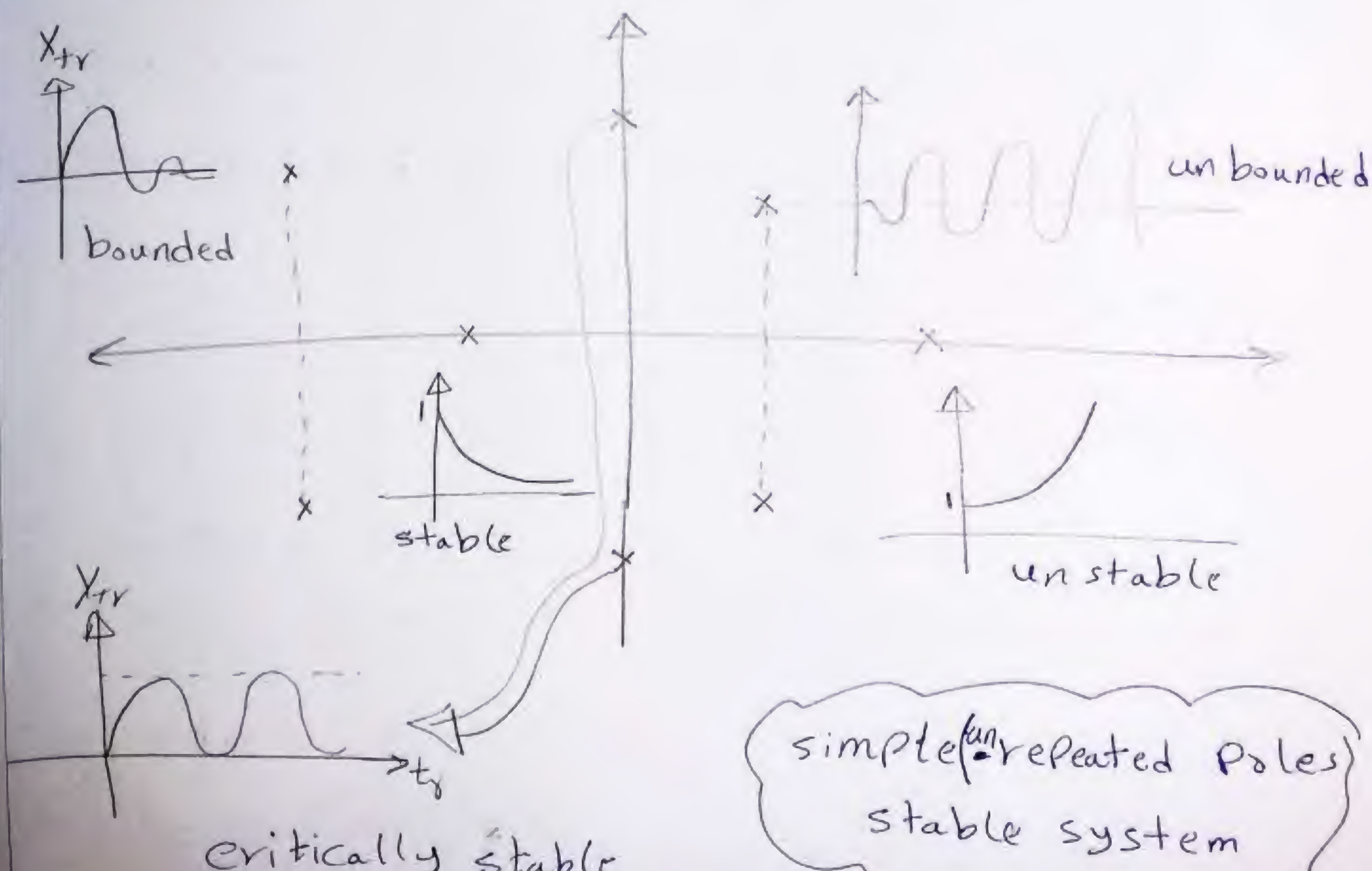
→ For n^{th} order system:-

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1+G(s)} = \frac{N(s)}{D(s)}$$

$$\boxed{D(s) = 1 + G(s) = a_0 s^n + a_1 s^{n-1} + \dots + a_{n-1} s + a_n = 0}$$

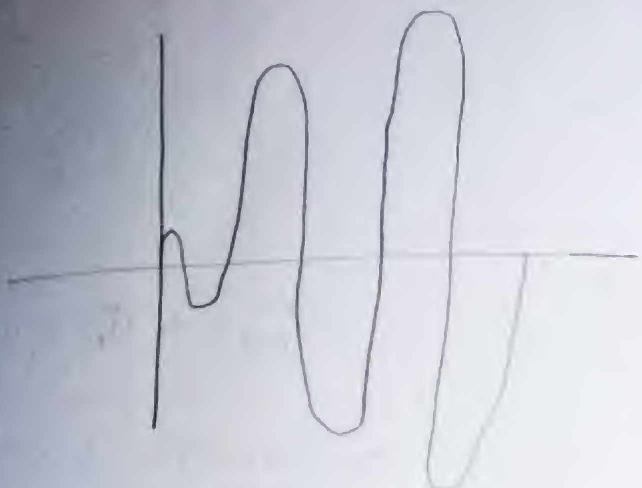
↳ charactristic eqn.

$$y(t) = Y_{ss}(t) + X_{tr}(t)$$

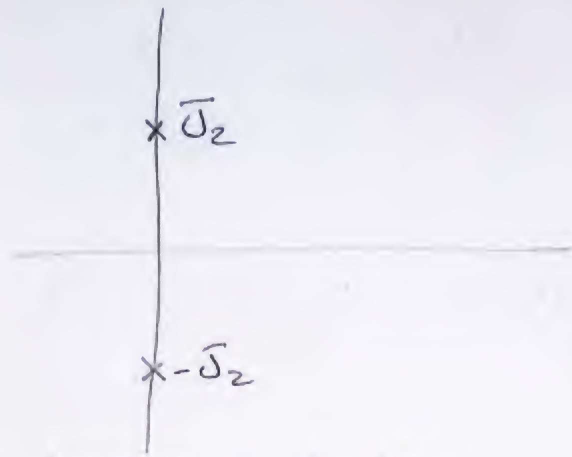


simple(un)repeated Poles stable system

← "Poles" على المحور الرأسي تكون إما "simple"
أو "repeated"



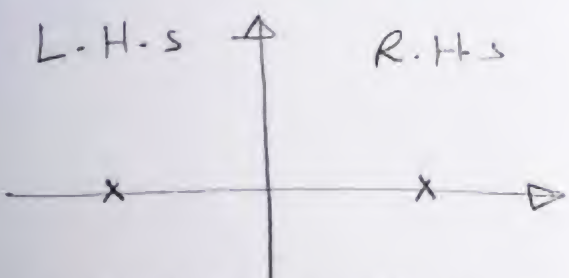
unstable system



repeated Poles

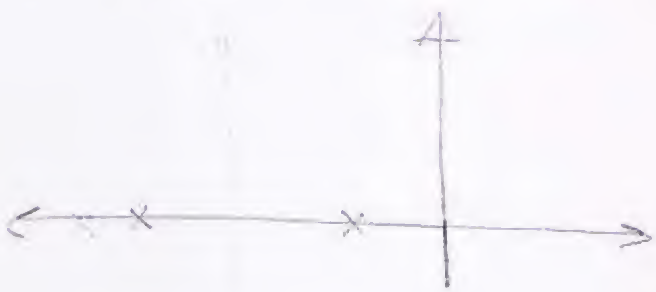
"simple Pole" ← يجعل "system" يكون "stable".

"repeated Pole" ← " " " "unstable"



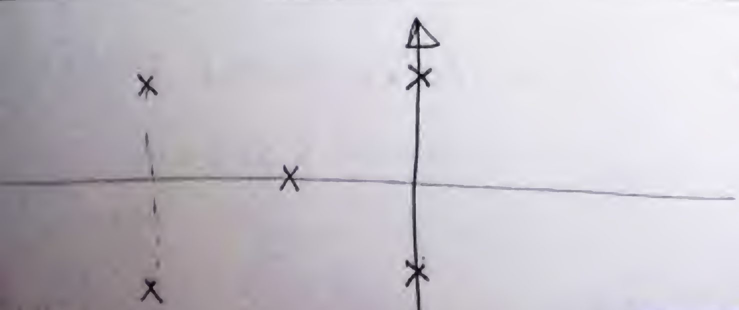
unstable system

لوجود "Poles" في R.H.S



Stable system

لعدم وجود "Poles" في



Critically stable

لوجود "Poles" على المحور الرأسي

"unstable system" لوجود "repeated poles"

تحدد ال (system) هل (stable) أو (unstable) بدون حساب الجذور.

* Routh - Harwitz array

معاملات chls eqn

$$
\begin{array}{c|ccc}
s^n & a_0 & a_2 & a_4 \\
\vdots & a_1 & a_3 & a_5 \\
\vdots & b_1 & b_2 & b_3 \\
\vdots & c_1 & c_2 & c_3 \\
 & \vdots & & \\
s^1 & \vdots & & \\
s^0 & a_n & &
\end{array}
$$

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1} \qquad c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1}$$

$$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1} \qquad c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1}$$

— يكون "system" ← stable إذا كان عناصر العمود الأول لها نفس الإشارة سواء + أو - ولو فيه إختلاف في الإشارة يكون unstable.

— عدد ال (Poles) ناحية (R.H.s) يساوي عدد المتغيرات في إشارة العمود الأول.

Examples $s^5 + 8s^4 + 2s^3 + 4s^2 + 2s + 4 = 0$

s^5	1	2	2	$\Rightarrow$	+1	2	2
s^4	8	4	4	$\xRightarrow{\div 4}$	+2	1	1
s^3	$\frac{3}{2}$	$\frac{3}{2}$		$\Rightarrow$	+1	1	
s^2	-1	1		$\Rightarrow$	- -1	1	
s^1	2			$\Rightarrow$	+ 2		
s^0	4			$\Rightarrow$	+ 4		

$\Rightarrow$ There are two sign changes in the elements of the 1st Column.

$\therefore$ The system is unstable:-

3 LH-s $\Leftarrow$ 2 R.H.s ← 5 Pole ←

Example $s^4 + s^3 + s^2 + s + 3 = 0$

نستبدل ٥ ← ϵ
لأنه لا يجب القسمة على ٥ .

sol

s^4	+ 1	1	3
s^3	+ 1	1	
s^2	+ $\overset{\epsilon}{0}$	3	
s^1	− $\frac{\epsilon - 3}{\epsilon}$		
s^0	+ 3		

$\frac{\epsilon - 3}{\epsilon} \longrightarrow$ لأنه ϵ صغيرة جدا .

← (unstable) لتغير الإشارة .

$\overbrace{\text{2 in R.H.S} \qquad \text{2 in L.H.S}}^{\text{4 poles}}$

← إذا وجدنا أنه عناصر الصف جميعها = صفر نحسب $A(s)$ ثم نأتي بتفاضل $\frac{dA(s)}{ds}$

x	x	x
x	x	x
o	o	o
x	x	x

x x x $\longrightarrow A(s)$

$A(s)$ من الصف الذي يسبق الصف الي عناصره = صفر .

Example $s^4 + s^3 - s - 1 = 0$

s^4	1	0	−1
s^3	1	−1	
s^2	1	−1	
s^1	0		
s^0	−1		

s^2: 1 −1 $\Longrightarrow A(s)$

s^1: 0 $\longrightarrow$ all zero row

auxillary eqn $A(s) = s^2 - 1 \Rightarrow \frac{dA(s)}{ds} = 2s$

- يوجد 1 Pole R.H.S لأن الإشارة تغيرت مرة واحدة.

- الصف اللى كله أصفار لابد أن يكون صف فردى.

Routh array

$$\begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \\ x & x & x \end{bmatrix} \begin{cases} \text{stable} \\ \text{unstable} \end{cases}$$

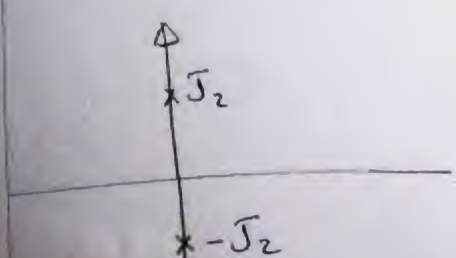
No all zero row

$$\begin{bmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \\ x & x & x \end{bmatrix} \begin{cases} \text{critically stable} \\ \text{unstable} \end{cases}$$

all zero row

$$\begin{array}{l} \\ A_1(s) \\ \\ A_2(s) \\ \\ \end{array} \begin{bmatrix} x & x & x \\ x & x & x \\ 0 & 0 & 0 \\ x & x & x \\ 0 & 0 & 0 \end{bmatrix} \text{unstable system}$$

$A(s) = 0$



sustaned (constant) oscillations

$s = Re.s + Im.s$

$= \sigma + j\omega$

Freq. of oscillations $= \omega$